Accounting for the polarizing effects introduced from non ideal quarter-wave plates in lidar measurements of the circular depolarization ratio

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[02].[Emerging lidar techniques, methodologies, and discoveries]

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General case: Identifying the polarizing behavior of an unknown optic

Mathematical formulation through **Müller** calculus of an optical assembly that consists of:

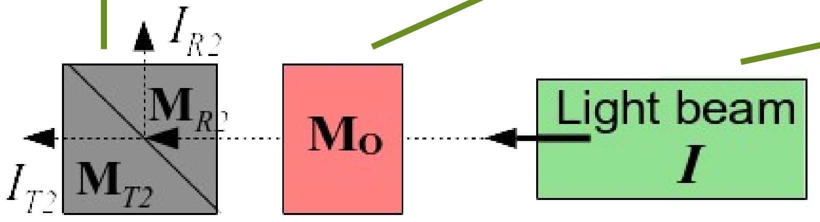
- A radiation beam **I** whose state of polarization is generally known
- A generally unknown array of optics ($\mathbf{M_O}$)
- A Polarizing Beamsplitter ($\mathbf{M_{PBS}}$)
- Two detecting channels ($\mathbf{I_S}$)

  * S = R or T for the reflected/transmitted paths

  * η_S is the channel opto-electronic amplification

Using the Müller formalism to describe the system

$$M_S = T_S \begin{pmatrix} 1 & D_S & 0 & 0 \\ D_S & 1 & 0 & 0 \\ 0 & 0 & Z_S & 0 \\ 0 & 0 & 0 & Z_S \end{pmatrix} \qquad M_O = T_O \begin{pmatrix} 1 & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \qquad I = I_O \begin{pmatrix} i \\ q \\ u \\ v \end{pmatrix}$$



$$\boldsymbol{I_S = \eta_S M_{PBS} M_O I}$$

The measured intensity in channel S (I_S) using the Bra-Ket formulation:

$$I_S = \eta_S T_S T_O I_O \begin{pmatrix} 1 + D_S m_{21} \\ m_{12} + D_S m_{22} \\ m_{13} + D_S m_{23} \\ m_{14} + D_S m_{24} \end{pmatrix} \begin{pmatrix} i \\ q \\ u \\ v \end{pmatrix}$$

$$I_S = \eta_S T_S T_O I_O \left[(1 + D_R m_{21}) i + (m_{12} + D_R m_{22}) q + (m_{13} + D_R m_{23}) u + (m_{14} + D_R m_{24}) v \right]$$

By dividing the measured I_R and I_T signals:

$$\frac{I_R}{I_T} = \frac{\eta_R T_R}{\eta_T T_T} \frac{(1 + D_R m_{21}) i + (m_{12} + D_R m_{22}) q + (m_{13} + D_R m_{23}) u + (m_{14} + D_R m_{24}) v}{(1 + D_T m_{21}) i + (m_{12} + D_T m_{22}) q + (m_{13} + D_T m_{23}) u + (m_{14} + D_T m_{24}) v}$$

Defining the measured quantity w:

$$w = \frac{I_R - \eta I_T}{D_T I_R - \eta D_R I_T} \quad , \quad -1 \leq w \leq 1$$

And the calibration factor η:

$$\eta = \frac{\eta_R T_R}{\eta_T T_T}$$

by substituting w:

The signal ratio can be reformed in a new expression. Here w is a measured quantity, i,q,u,v are the emitted Stokes vector elements and $m_{i,j}$ the M_O elements:

$$w q m_{12} + w u m_{13} + w v m_{14} + i m_{21} + q m_{22} + u m_{23} + v m_{24} = -wi$$

General case: Identifying the polarizing behavior of an unknown optic

Assuming:

- n different (but known) states of polarization emitted
- a measurement of the signals ratios per emitted state n
- a known calibration factor η
- seven unknown $m_{i,j}$ elements

The following n x 7 system of linear equations can be formed:

$$w\,q\,m_{12} + w\,u\,m_{13} + w\,v\,m_{14} + i\,m_{21} + q\,m_{22} + u\,m_{23} + v\,m_{24} = -wi$$

$$\begin{pmatrix} w_1 q_1 & w_1 u_1 & w_1 v_1 & i_1 & q_1 & u_1 & v_1 \\ w_2 q_2 & w_2 u_2 & w_2 v_2 & i_2 & q_2 & u_2 & v_2 \\ \vdots & & & & & \ddots & \\ w_n q_n & w_n u_n & w_n v_n & i_n & q_n & u_n & v_n \end{pmatrix} \begin{pmatrix} m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \end{pmatrix} = \begin{pmatrix} -w_1 i_1 \\ -w_2 i_2 \\ \vdots \\ -w_n i_n \end{pmatrix}$$

$A_{n,k} \quad * \quad X_{k,1} = B_{n,1}$

The system can be solved for the seven $m_{i,j}$ elements with the **linear least squares method**:

$$X = \left(A^T A\right)^{-1} A^T B$$

In order for the system to be determined or overdetermined the rank of the coefficient matrix A must be seven

<u>Some notes:</u>

- Only the upper two rows of M_O can affect the measured intensities I_R and I_T when a PBS is used as a linear analyzer
- The calibration factor of the system must be measured independently (e.g. with a Δ90 calibration, Freudenthaler 2016)

<u>Special cases:</u>

- If some $m_{i,j} = 0$, they can be omitted along with the respective column of A --> lower system rank
- If one of the q, u, v elements is zero for all n states then the affected columns of A and rows of X must be omitted

  * e.g. if v = 0 for each n then the k = 3, 7 columns and rows must be omitted --> not possible to get m_{14} and m_{24}

Special case: The receiver array of optics is a rotated retarder

The Muller matrix of a rotated retarder (taken from Freudenthaler 2016):

$$M_O = T_O \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-s_{2\varphi}^2(1-c_O) & s_{2\varphi}c_{2\varphi}(1-c_O) & -s_{2\varphi}s_O \\ 0 & s_{2\varphi}c_{2\varphi}(1-c_O) & 1-c_{2\varphi}^2(1-c_O) & c_{2\varphi}s_O \\ 0 & s_{2\varphi}s_O & -c_{2\varphi}s_O & c_O \end{pmatrix}$$

where $s_{2\varphi} = \sin(2\varphi)$, $c_{2\varphi} = \cos(2\varphi)$, $s_O = \sin\Delta_O$, $c_O = \cos\Delta_O$

Some Rotated Retarders:

- **Ideal Quarter-wave plates** (QWP) when rotated are a special case of rotated retarder with $|\Delta_O| = 90^o$
- The same is true for **ideal Half-wave plates** (HWP) with $|\Delta_O| = 180^o$
- A **non ideal QWP or HWP** which introduces a retardation error from its ideal value is also a rotated retarder

A rotated retarder is not introducing diattenuation. This is generally expected from QWP and HWP even if their retardation is not ideal

Simplification of the system from rank 7 to rank 2 when some information on M_O is already known:

$$\begin{pmatrix} \cancel{w_1 q_1} & \cancel{w_1 u_1} & \cancel{w_1 v_1} & \cancel{i_1} & q_1 & u_1 & \cancel{v_1} \\ \cancel{w_2 q_2} & \cancel{w_2 u_2} & \cancel{w_2 v_2} & \cancel{i_2} & q_2 & u_2 & \cancel{v_2} \\ \vdots & & & & & \ddots & \\ \cancel{w_n q_n} & \cancel{w_n u_n} & \cancel{w_n v_n} & \cancel{i_n} & q_n & u_n & \cancel{v_n} \end{pmatrix} \begin{pmatrix} \cancel{m_{12}} \\ \cancel{m_{13}} \\ \cancel{m_{14}} \\ \cancel{m_{21}} \\ m_{22} \\ m_{23} \\ \cancel{m_{24}} \end{pmatrix} = \begin{pmatrix} -w_1 i_1 \\ -w_2 i_2 \\ \vdots \\ -w_n i_n \end{pmatrix}$$

The M_O retardation:

$$\Delta_O = 2k\pi \pm \arccos\left(m_{22} + \frac{m_{23}^2}{1-m_{22}}\right)$$

The M_O rotation:

$$\varphi = \frac{k\pi}{2} + \frac{1}{2}\arctan\left(\frac{1-m_{22}}{m_{23}}\right)$$

The simplified system:

$$\begin{pmatrix} q_1 & u_1 \\ q_2 & u_2 \\ \vdots & \vdots \\ q_n & u_n \end{pmatrix} \begin{pmatrix} m_{22} \\ m_{23} \end{pmatrix} = \begin{pmatrix} -w_1 i_1 \\ -w_2 i_2 \\ \vdots \\ -w_n i_n \end{pmatrix}$$

m_{22} and m_{23} can be calculated by using at least n = 2 different states of polarization --> then **Δ_O and φ** can be analytically retrieved from them

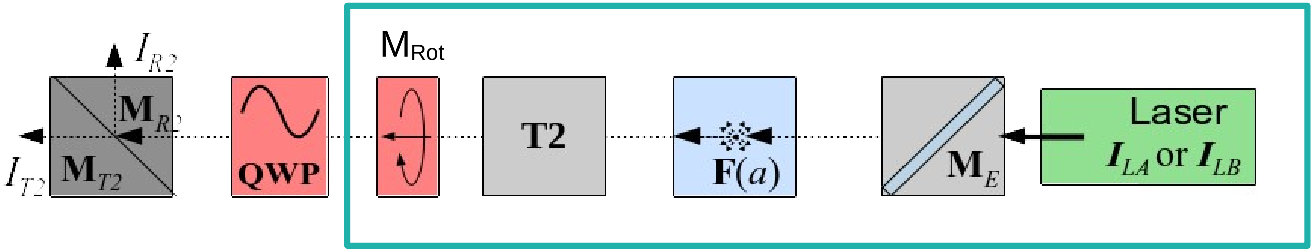
Application on the EVE lidar system

The EVE lidar:

- EVE is a 2 telescope x 2 laser lidar system, built to validate the AEOLUS satellite mission
- Both laser beams can be recorded simultaneously in both receivers
- Laser A and B emit linearly polarized light
- A QWP is normally placed at 45^o in front of Laser B to convert the polarization from linear to circular
- **The two QWP of EVE were suspected to be deviating from ideal behavior**

The measurements were performed on field with EVE using both of its lasers and were repeated in the laboratory with an ellipsometer to verify the retrievals

The 2nd receiver of EVE where circularly polarized light is normally detected. **The QWP in front of Laser B was removed for the tests**



The manual rotator M_{Rot} rotated at (-45^o, 0^o, +45^o) provides the different states of polarization needed for the method. The telescope T_2 and the emission optics M_E do not alter the state of polarization. Calculations are performed in aerosol-free regions where F(α) is known (molecular backscatter)

A Monte Carlo error simulation is performed to handle error propagation

Retrieved parameters for the QWP in the 2nd receiver. Here ω and ε are the retardation and rotation errors of the QWP from the ideal values of 90^o and 45^o, respectively.

	m_{22} (10^{-2})	m_{23} (10^{-2})	Δ_Q (°)	φ (°)	ω (°)	ε_{QWP} (°)
Laser A	8.4 ± 1.1	1.3 ± 1.2	85.2 ± 0.6	44.7 ± 0.3	-4.8 ± 0.6	-0.3 ± 0.3
Laser B	8.6 ± 0.7	2.3 ± 0.9	85.1 ± 0.4	44.4 ± 0.2	-4.9 ± 0.4	-0.6 ± 0.2
LEM	8.2 ± 1.9	2.3 ± 2.0	85.3 ± 1.1	44.3 ± 0.6	-4.7 ± 1.1	-0.7 ± 0.6

Main points

- A technique was developed that enables the calculation of individual Müller matrix elements of a a generally unknown optic in a lidar system
- The technique was applied on a QWP that was suspected to be malfunctioning directly on the EVE lidar system and also in the laboratory
- Both the on-field and the laboratory tests showed a similar absolute retardation error of ~5^{o}. The respective rotation error was small in the order of ~0.5^{o} for all the tests
- The error of the method was determined using a Monte Carlo error simulation taking into account the uncertainty of the recorded signals, the calibration factor, and the emitted Stokes vector
- In the next step the effect of the non-ideal QWP will be included and properly accounted for within the GHK correction (Freudentaler 2016) that is applied to the EVE depolarization ratio profiles